Edexcel International AS/A Level Mathematics

Teaching and Learning Strategies: Module 2

First teaching in 2018, first assessment 2019

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Session agenda

10:00	Welcome & introductions
10:10	Mathematical facts
10:40	Mathematical processes
10:50	Problem solving skills
11:10	Mathematical reasoning
11:20	Teaching strategies
11:45	Support, resources and final questions
12:00	Finish

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Aims and objectives

Delegates will:

- learn from analysis of how students have performed in examinations to identify those areas of learning which students have found most challenging

- be introduced to a range of teaching and learning strategies particularly applicable to mathematics

- discuss strategies for optimising the learning of students in number, algebra, geometry and statistics

- have an opportunity to share best practice of what has worked well with students studying these specifications.

Polls to get to know the delegates

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Introduction

- Strategies for improving examination performance:
    - what facts students should know and how to make sure they do
    - what processes students should know and how to make sure they do
    - what skills should students have to answer problem solving questions
    - what skills should students have to answer proof and argument questions.

- Resource-based learning: using Edexcel's suite of resources to improve performance in mathematics examinations.

Mathematical facts

A mathematical fact could be:

- a formula – for example, the formula for the solution of a quadratic equation
- a reason – for example, the angle in a semicircle is a right angle
- a relationship – for example, given $x^2 = a$ then $x = \pm\sqrt{a}$

Mathematical facts

Activity 1

(a) Write down some formulae that students need to know for Higher Tier International GCSE.

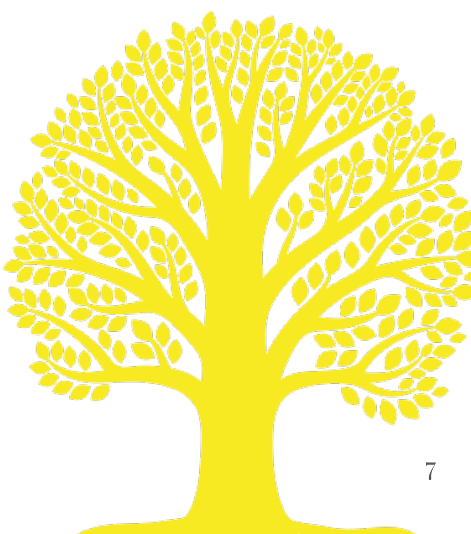
Aim to get a minimum of 5.

Do students at Higher Tier need to <u>learn</u> the Sine Rule?

Do students at Higher Tier need to <u>learn</u> the Cosine Rule

Use the Poll for Activity 1 to respond.

Responses are shared anonymously.



Mathematical facts

Some formulae:

- Area of a circle in terms of the radius
- Circumference of a circle in terms of the diameter
- Speed = Distance/Time
- sin *A* = opp/hyp
- Sum of the interior angles of an *n*-sided polygon = (2*n* – 4) right angles or (n-2) x 180^0
- Area of a parallelogram = *bc* sin*A*

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Mathematical facts

Some formulae and their associates:

- radius of a circle given its area
- diameter of a circle given its circumference
- Distance = Speed × Time, Time = Distance/Speed
- Opp = hyp × sin *A*, hyp = opp/sin *A*

Mathematical facts

Some mathematical reasons:

- Correct language is very important:
- ‘The sum of the interior angles of a triangle is 180^{o}’
- ‘The angle subtended at the centre of a circle by a chord is twice the angle subtended at the circumference by the same chord’
- ‘alternate angles are equal’

Mathematical facts

Some mathematical relationships:

If $ax = b$ then $x = b/a$

The length of a side of a square is equal to the square root of its area.

If y = f(x), then x = f^{-1} (y)

$(2x + 4y)^2 = 4(x + 2y)^2$

Mathematical facts

Recall of mathematical facts requires a good long term memory for mathematics.

Improving long term memory in mathematics:

- Improving knowledge of formulae
- Improving knowledge of reasons
- Improving knowledge of relationships

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Mathematical facts

Improving long term memory in mathematics:

- Improving knowledge of formulae
- Enforce learning of formulae even if in the formula book (or sheet)

e.g. the expansion of sin (A + B)

Expect students to learn alternative versions of standard formulae (that may be in the formula book/sheet).

e.g. the alternative version of the cosine rule i.e. the cos A = form

Mathematical facts

Improving long term memory in mathematics:

Improving knowledge of formulae:

- Do this by having short tests very often (you may have to convince some students that learning extra material, some of which is in the formula book/page, will lead to better results).

- You could build in such tests into your Scheme of Work (SOW) and/or lesson plans.

Mathematical facts

Improving long term memory in mathematics:

Improving knowledge of reasons (mainly International GCSE):

- Do this by having short tests very often (you may have to convince some students that they need to use precise language)
- You could build in such tests into your Scheme of Work (SOW) and/or lesson plans.
- You could insist students give reasons even when not asked for – for example in calculating angles.

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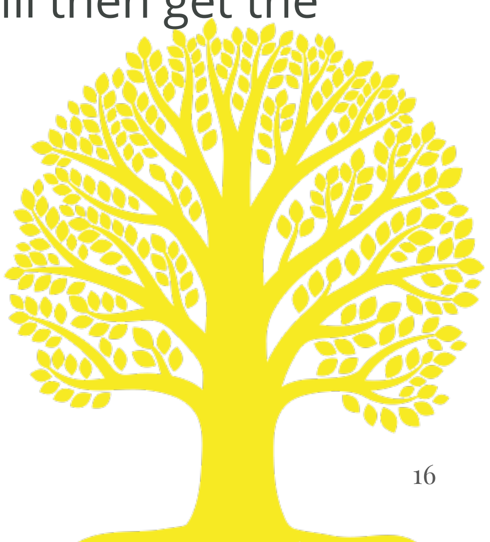
Mathematical facts

Improving long term memory in mathematics:

Improving knowledge of relationships:

- Do this by having short tests very often.
- You could build in such tests into your Scheme of Work (SOW) and/or lesson plans.
- What can happen is that some of these relationships will then get the status of settled facts.



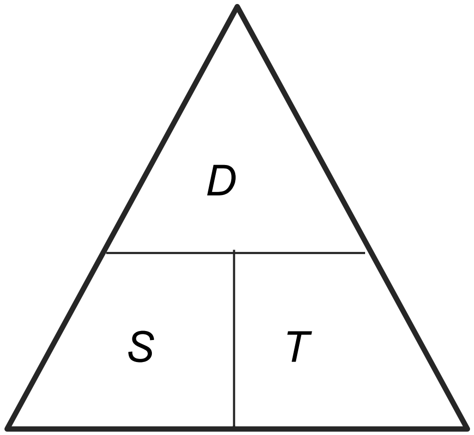
Mathematical facts

Improving long term memory in mathematics:

Improving knowledge of relationships:

Many teachers use a visual cue for relationships.

The most well known being:



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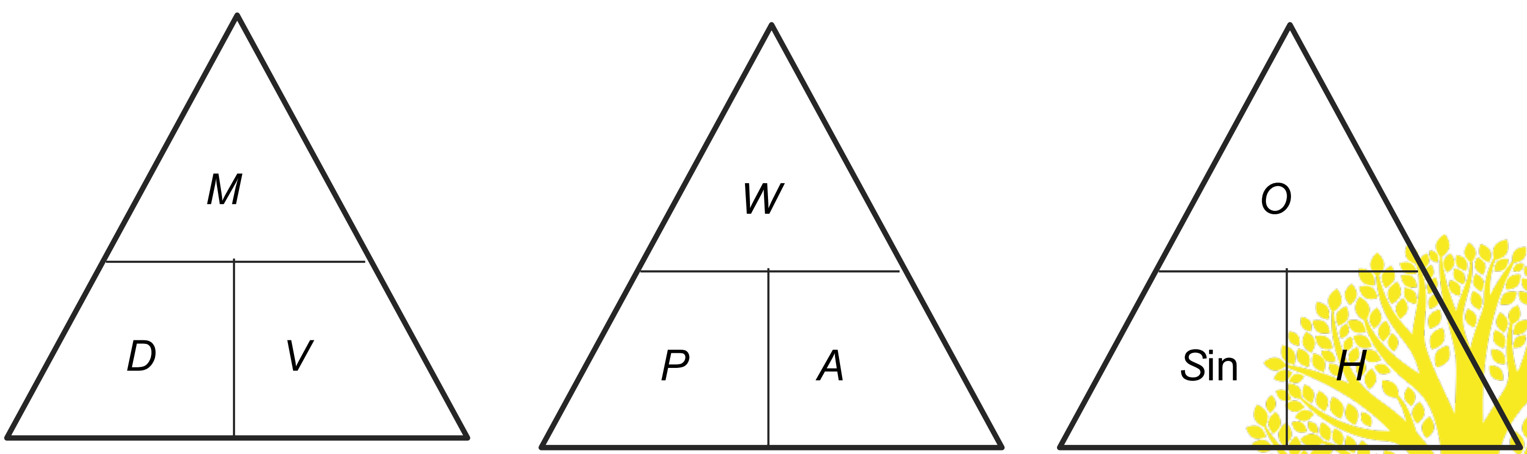
Mathematical facts

Improving long term memory in mathematics:

Improving knowledge of relationships

This can be adapted as below:



Mathematical facts

Improving long term memory in mathematics:

Improving knowledge of relationships:

In some cases this can be done by standard memory techniques either using ‘sound’ or ‘visual’.

The most well-known sound ones are

SOCAHTOA for trig ratios
SSS and OSA for simultaneous equations

and of course the best known visual ones were shown on the previous slide.

Mathematical processes

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Mathematical processes

A mathematical process consists of a sequence of steps to achieve an answer:

- Solving a pair of simultaneous equations
- Performing an integration by substitution
- Calculating the regression line on a set of bivariate data.

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Mathematical processes

Many of these themselves consist of a series of shorter processes.

For example, solving a pair of simultaneous equations by elimination:

- Multiply both sides of each equation by suitable numbers to make the coefficients of one variable the same
- Add or subtract the equations (i.e. add/subtract both sides) to eliminate one variable
- Solve the resulting equation for that variable
- Substitute back and solve for the other variable.

Mathematical processes

As the previous slide shows, there are many places in the sequence of simpler processes where students can go wrong.

So it is very important that good prior knowledge plays a leading role in SoWs.

Students should be tested on prior knowledge and remedial action taken if necessary.

Mathematical processes

Similarly for IAL, most processes depend on others being carried out correctly.

For example: $$\int \frac{4x^2+1}{2\sqrt{x}}\,\mathrm{d}x$$

requires the integrand to be written as two terms before direct integration.

In module 1 we saw the errors that substantial numbers of students made when they tried to do that.

Mathematical processes

Activity 2

What prior knowledge is required to solve a pair of simultaneous equations where one is quadratic and one is linear?

For example:

$$y = 2x + 5$$

$$x^2 + y = 2$$

Or the more challenging:

$$2x + 3y = 13$$

$$\boldsymbol{x^2 + 2y^2 = 22}$$

Use Chat to make your suggestions.

Mathematical processes

Activity 2

Rearranging $ax + by = c$ to make x or y the subject.

Substituting an expression of the form $\dfrac{c - bx}{a}$ in a quadratic in 2 variables to make a quadratic in 1 variable.

Simplify to a 3 term quadratic (3TQ) – will often require expanding brackets.

Solve the 3TQ by factorisation, formula or completing the square.

Mathematical processes

Improving long term memory in mathematics:

Improve knowledge of standard procedures

Again, short tests can be useful.

Items such as: Solve $x(x - 4) = 2x^2$

Solve $\sin (2x + 40)^{\circ} = 0.5$ (i.e. where mistakes are common)

Find an expression equal to $(\sin x + \cos x)^2$

i.e. where students have yet to make connections between trig functions.

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Problem solving skills

Problem solving

A reminder of what a problem is:

> 'a task where the solution is not immediately attainable and there is no obvious algorithm for the student to use.'

So the solution of simultaneous equations (one linear, one quadratic) would not be a problem unless…

https://www.popularmechanics.com/science/g2816/5-simple-math-problems/

Problem solving

A reminder of what makes a task ‘hard’:

- the complexity of elements of knowledge or task – linked to the expectation of the content standards of the qualification level
- the number of steps/linkages involved in a response
- the level of familiarity/prior knowledge students may have on the content or procedures required; the procedure is not routine and may require any adaptation or application.
- the predictability of a question – from series to series – how familiar the question is over time.

Problem solving

- Module 1 looked at the idea of familiarity and at the number of steps required to complete a task.

- Unfamiliar questions just turn out to be harder.

- Questions which require a large number of steps just turn out to be harder.

Problem solving

Similarly, in module 1 when looking at the characteristics of a problem:

- **A**. There is little or no scaffolding: little guidance given to the student beyond a start point and a finish point. Questions do not explicitly state the mathematical process(es) required for the solution.
- **B.** There is a need for multiple representations, such as the use of a sketch or a diagram as well as calculations.
- **C.** The information is not given in mathematical form or in mathematical language; or there is a need for the results to be interpreted or methods evaluated, for example, in a real-world context.
- **D.** There is a variety of techniques that could be used.
- **E.** The solution requires understanding of the processes involved rather than just application of the techniques.

Problem solving

An example which displays many of these characteristics was Q22 of Jan 2020 4MA1 1H.

22 Triangle HJK is isosceles with $HJ = HK$ and $JK = \sqrt{80}$

H is the point with coordinates $(-4, 1)$
J is the point with coordinates $(j, 15)$ where $j < 0$
K is the point with coordinates $(6, k)$

M is the midpoint of JK.
The gradient of HM is 2

Find the value of j and the value of k.

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Problem solving

An example which displays many of these characteristics was Q22 of Jan 2020 4MA1 1H.

Activity 3

Look at the question and the mark scheme.

Then complete the poll.

Results will be shared anonymously

Problem solving

One possible set of responses to this problem:

A No guidance. (It could have started with, for example, write down the gradient of JK.)

B No diagram was given. This was a problem cast in geometrical form but most likely had to be done using algebra.

C This meant finding suitable algebraic equations using geometrical properties. (If they produced and solved a quadratic equation for *k*, say, they had to reject one of the values) so there was an element of interpretation.

D It's clear from the complexity of the mark scheme that this was the case.

E ....

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Problem solving

One possible set of responses to this problem:

1. The task was complex and the techniques required advanced, with some approaches using Pythagoras in a sophisticated way together with simultaneous equations, only one of which was linear.

2. There were a large number of steps, depending on method – in some cases ten or more.

3. No problem like this had been set before by Edexcel.

4. No problem like this had been set before by Edexcel and no one could have predicted a question of this type (but maybe of this demand).

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Problem solving

How should students attempt problems of this level of complexity?

Approaches could vary:

In this case the first would be to draw a good, properly labelled diagram – as should be done in any geometrical problem where a diagram is not given.

Secondly, a student would have to draw on basic knowledge of the properties of isosceles triangles.

Thirdly, find suitable equations using that knowledge – that is translating to an algebraic form.
Since there are two unknowns, students should know they need two equations.

Numbering the equations should also help keep track of progress through the problem.

Problem solving

23 Boris has a bag that only contains red sweets and green sweets.

Boris takes at random 2 sweets from the bag.

The probability that Boris takes exactly 1 red sweet from the bag is $\frac{12}{35}$

Originally there were 3 red sweets in the bag.

Work out how many green sweets there were originally in the bag.
Show your working clearly.

4MA1 June 2019 2H

- This requires an algebraic approach.
- Clearly define g as the number of green sweets originally in the bag.
- Know that there are 2 cases to consider RG and GR or even better 2RG.
- Realise that it's sampling without replacement.

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Problem solving

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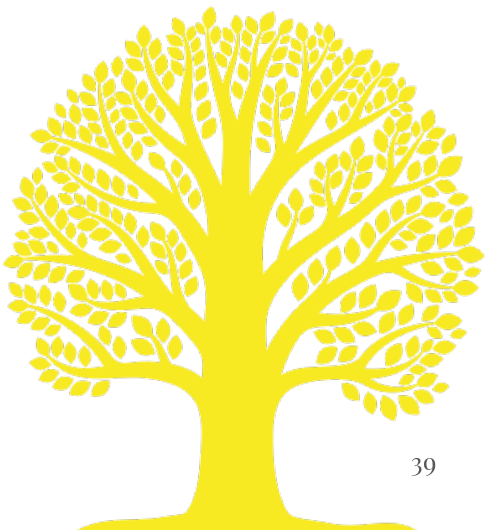
Work out how many green sweets there were originally in the bag.
Show your working clearly.

- <u>Translate</u> into algebraic form.
- <u>Solve</u> the resulting equation.
- <u>Interpret</u> the solution(s).
- And in this case <u>numerically check</u> the answer.



Problem solving

23 Boris has a bag that only contains red sweets and green sweets.

Boris takes at random 2 sweets from the bag.

The probability that Boris takes exactly 1 red sweet from the bag is $\frac{12}{35}$

Originally there were 3 red sweets in the bag.

Work out how many green sweets there were originally in the bag.
Show your working clearly.

So, let g be the number of green counters originally in the bag.

Therefore there are $g + 3$ counters originally in the bag.

Therefore: $$\frac{g}{g+3} \times \frac{3}{g+2} \times 2 = \frac{12}{35}$$

$$12g^2 - 150g + 72 = 0$$

$$g = 12 \text{ or } g = 0.5$$

Mathematical reasoning

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Mathematical reasoning

Mathematical reasoning for International GCSE was described in module 1.

There are three main areas:

- Number – fractions, factors, surds and recurring decimals
- Algebra – algebraic proofs of identities involving manipulation
- Geometry – supplying supporting reasons for calculations.

Mathematical reasoning

Mathematical reasoning for International GCSE was described in module 1.

Number – the key point is to demonstrate all steps.

e.g. for $\left(\sqrt{3}+\sqrt{12}\right)^2$

students should either show the expansion (without simplification) or use

$$\sqrt{12}=\sqrt{4}\times\sqrt{3}=2\sqrt{3}$$

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Mathematical reasoning

Mathematical reasoning for International GCSE was described in module 1.

Algebra – depending on the form, all steps should be shown.

e.g. Prove algebraically that the difference between squares of consecutive odd numbers is a multiple of 8

where a complete final expression is not shown.

Showing all steps is even more important when there is an expression to derive

e.g. The dimensions in cm of a cuboid are $(x + 3)$, $(x - 1)$ and $(2x - 3)$, prove that the volume, V cm^3 of the cuboid is given by

$$V = 2x^3 + x^2 - 12x + 9$$

Mathematical reasoning

Mathematical reasoning for International GCSE was described in module 1.

Geometry – usually as reasons to accompany a geometrical calculation.

Mathematical reasoning

Mathematical reasoning for IAL builds on the ideas of International GCSE.

Proof of trigonometrical formulae appears as well as the use of the formal □

For example, this proof from a practice paper Pure 4:

Prove that $\sin 3t$ □ $3 \sin t - 4 \sin^3 t$

For this, acceptable steps which can be quoted include the sin (A + B) expansion (in the formulae book) and the sin 2A and cos 2A expansions (stated in the specification as learned expansions).

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Teaching strategies

Teaching strategies

We will look at:

- some specific ideas which come from examiner reports and recommendations

- some generic ideas for the classroom which come from trainers and education experts in the Pearson group

- some ideas on improving learning which come from Pearson-inspired research.

Teaching strategies

Specific ideas

- Encourage students to check answers
- Use the original equation(s) to check solutions of equations
- Use a suitable value of the variable to check expansions

  e.g. to check if $(2x - 1)(x - 2)(2x + 1) = 4x^3 - 8x^2 - x + 2$
  try $x = 1$

- If the student has worked methodically, it can also help locate when any errors first occurred. This can work well in proofs of trig identities.
- Check indefinite integration by differentiating the answer.

Teaching strategies

Specific ideas

- Encourage students to work line by line down the page

- When solving equations number the lines and refer to them,
  e.g. ‘Substitute (4) in (2)’ or ‘(2) + (4)’

  Similarly with proving trig identities

  Similarly with integrals by substitution… and many others.

Teaching strategies

Specific ideas

Harder problems on the International GCSE probability specification often involve:

- selecting at random without replacement – so the denominators of the relevant probabilities when written as fractions will decrease by 1
- having several ways in which a selection can be made
- teach students to write out the selections first; then multiply the probability of one of these by the number of different selections.

Teaching strategies

Specific ideas

Having several ways in which a selection can be made:

e.g. There are 4 red, 3 green and 2 yellow beads in a box.
Saba takes 3 beads at random from the box.

What is the probability they are all different colours?

The required answer is 6 times the probability of R then G then Y.

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Teaching strategies

Teaching problem solving

- Encourage students to plan what they are going to do.
- Depending on class culture this can be a good exercise in ‘thinking aloud’.
- Students should observe their teacher doing the same thing.
- It is very important that teachers display their thinking processes rather than just present solutions.

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Teaching strategies

Teaching problem solving – an example

Problem solving exemplar 1

A circle, centre C (1, 1), touches both axes, as shown in Fig. 1. AB is a tangent to the circle. The triangle OAB is isosceles.

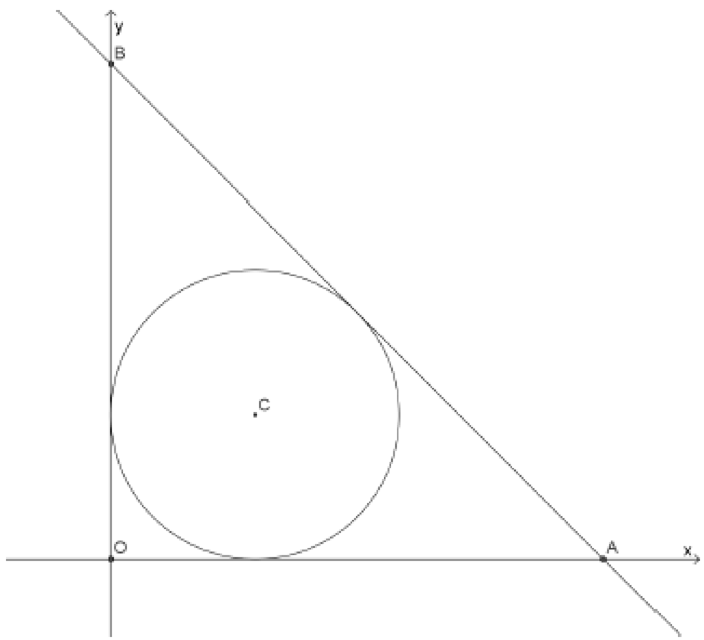

Fig. 1

Find the equation of AB, giving your answer in exact form. [8]

Make a plan

Teaching strategies

Teaching problem solving – an example

Problem solving exemplar 1

A circle, centre C (1, 1), touches both axes, as shown in Fig. 1. AB is a tangent to the circle. The triangle OAB is isosceles.

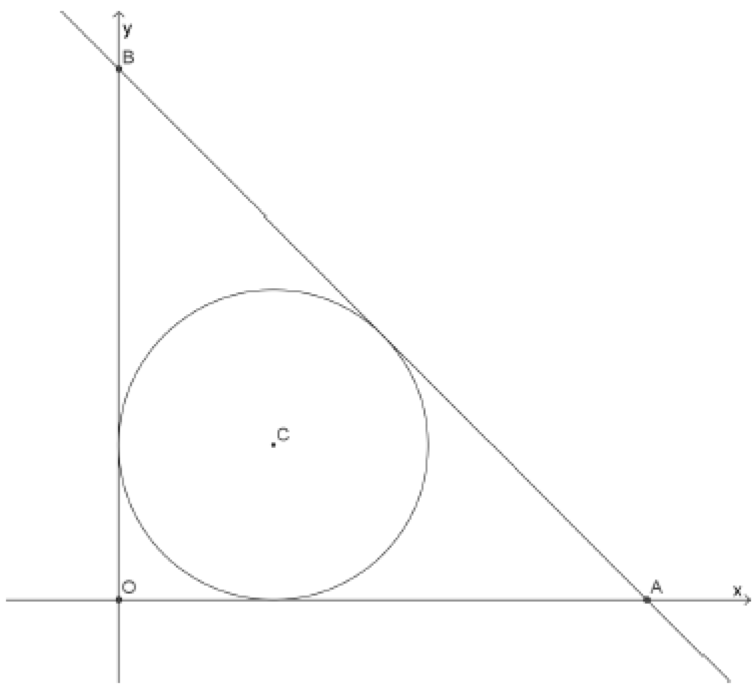

Fig. 1

Find the equation of AB, giving your answer in exact form. [8]

Make a plan

Use y = mx + c

Find m (easy)
Find c (harder) – extend OC to the line AB and use Pythagoras to find OB (and hence c)

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Teaching strategies

Think back to the previous example

Features for successful problem solving in examinations

- Having an outline plan
- Knowing the subject matter really well (to make the plan work)
- Carrying out the plan – best to show working line by line
- Trying to anticipate whether the plan is going in the right direction
- Asking if the answer is reasonable and whether all the information has been used.

Teaching strategies

Some generic points:

*Across most countries, a strong disciplinary climate is consistently and robustly associated with better performance.**

Being clear about expectations in terms of punctuality, work completion and exam/test preparation is very important.

Share objectives and the timescale with the students so they know what (approximately) is going to be done when (approximately).

Give feedback often and make it supportive – suggest where and how the student can improve.

* Pisa available from https://www.oecd-ilibrary.org/docserver/9789264039520-en.pdf?expires=1576523109&id=id&accname=guest&checksum=8561ED821869803C03DC158D296E6EDB

Teaching strategies

Some generic points:

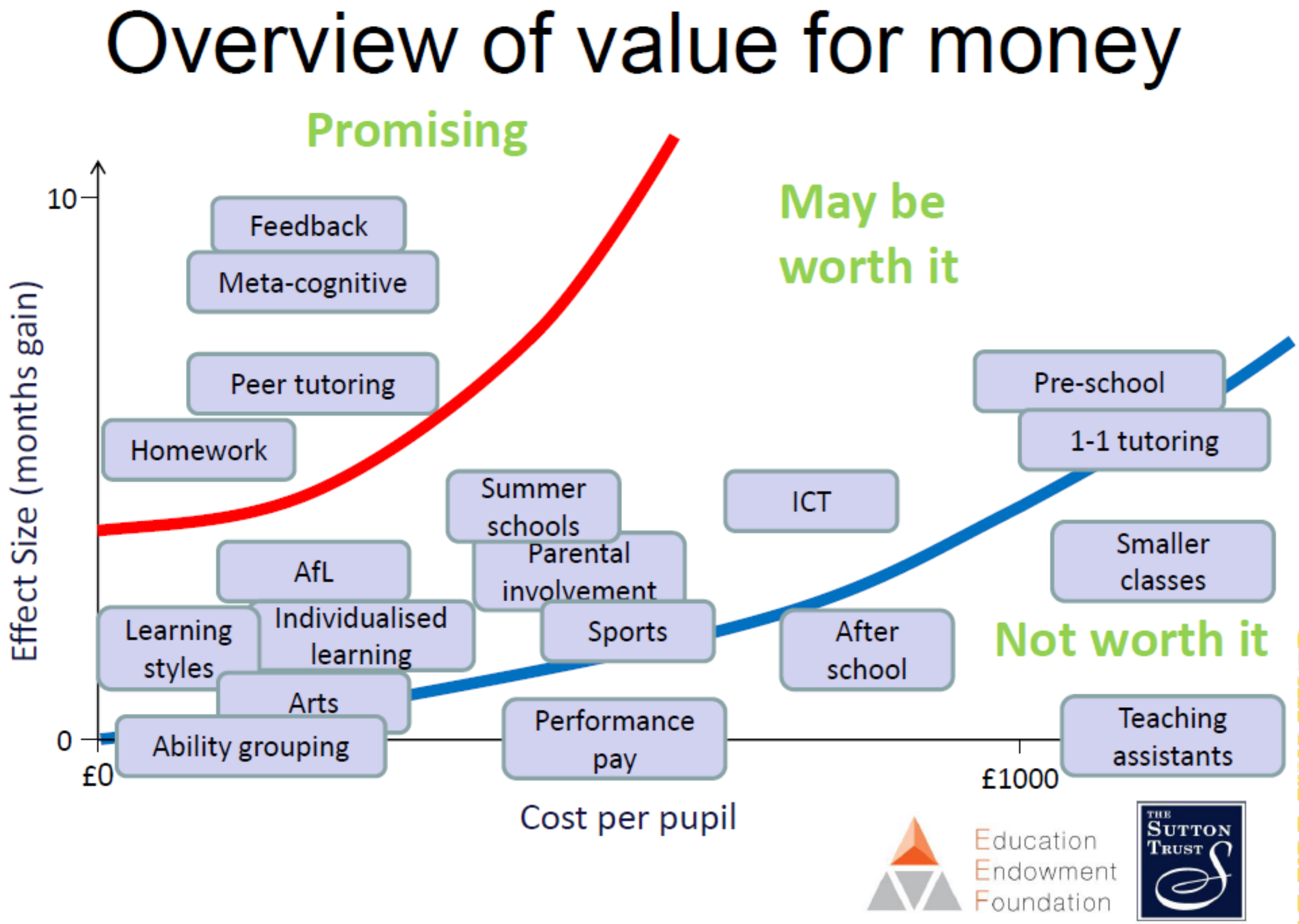
- Insist that students learn facts and formulae even if in the formula book – it will increase their fluency when it comes to solving problems.
- Model the solution of standard questions; set similar questions and review. The aim is that all students understand the subject matter.
- For a ‘new’ topic, show how it fits in with previous topics – review and revise if necessary.
- Give tasks for which there is more than one approach.
- Share responses with all members of the class.

Use ICT as a time saver and expect students to do the same.

Teaching strategies

Costs in 2013

Improving learning



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Teaching strategies

What seems to work:

Metacognitive learning

- This should be a school-wide process
- It contributes to the development of transferable skills independent learning in students

The need for transferable skills

In recent years, higher education institutions and employers have consistently flagged the need for students to develop a range of transferable skills to enable them to respond with confidence to the demands of undergraduate study and the world of work.
The Organisation for Economic Co-operation and Development (OECD) defines skills, or competencies, as ‘the bundle of knowledge, attributes and capacities that can be learned and that enable individuals to successfully perform…’

Metacognitive learning corresponds to the <u>cognitive</u> domain.

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Teaching strategies

What seems to work:

Metacognitive learning

- Students **plan** how to approach a given task
- Students **monitor** their comprehension of the task
- Students **evaluate** their progress towards the completion of the task
- Students **reflect** on how they completed the task.

These are similar to the skills we want students to have when solving problems in mathematics.

Teaching strategies

What seems to work:

Feedback

- Written in terms of ‘how to improve’ when returning marked homework…
  …and then following up to see if they have

- Showing appreciation of ideas and answers in class even if they are wrong and trying to get a student to see why they are

- Test and mock exam marking: tell them **how** they got it wrong
tell/ask **why** they got it wrong
tell/ask **how** can they get it right.

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Support from Pearson

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Support from Pearson

What seems to work:

Feedback from tests and exams

Ways in which valid and reliable questions can be obtained include:

- examWizard
- ResultsPlus
- both free to Edexcel centres.

Support from Pearson

Activity 5

Please complete the poll for examWizard and ResultsPlus.

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examWizard

- A free tool for teachers which helps you make quick homework assignments, topic tests and mock exams.
- Questions tagged against unit, topic and assessment objective or simply choose a whole past paper.
- Use existing mark schemes for accurate marking.
- Use examiner reports for insight.
- Most recent exam content available sooner.
- Use the results to understand where students need more support, informing teaching strategies.

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ResultsPlus

- Free online results analysis tool for teachers.
- Provides a detailed breakdown of student performance in Pearson Edexcel exams.
- Identify topics and questions where the student could benefit from further learning and inform teaching strategies and approaches.
- Benchmark your school's performance against other Pearson Edexcel schools in your country.
- Not just a post-results tool: Mock exam results can also be fed into the system to produce analysis.
- Find student results analysis from their previous Pearson Edexcel school.
- ResultsPlus Direct gives your students access to their final grades and performance breakdown, wherever they are.
- Schools can sign up for a free ResultsPlus account in just a few quick and easy steps: https://qualifications.pearson.com/en/support/Services/ResultsPlus.html
- This clip shows how mock papers are marked at the centre: https://www.youtube.com/watch?v=2bpOI5sHzjA

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Support from Pearson

Using Pearson support to enhance student outcomes

Many international centres already use ResultsPlus as a tool to analyse the performance of their students.

For a unitised course this can be used usefully as a teaching resource:

- Use Access to Scripts Service (ATS) to look at the actual answers of a student.
- (Decide whether to ask for a re-mark.)
- Use ResultsPlus for one or more student(s) to compare their performance with the general entry.
- Use examWizard to produce targeted practice material for resit student(s).

All these features are free to Edexcel centres

Support from Pearson

Planning the course

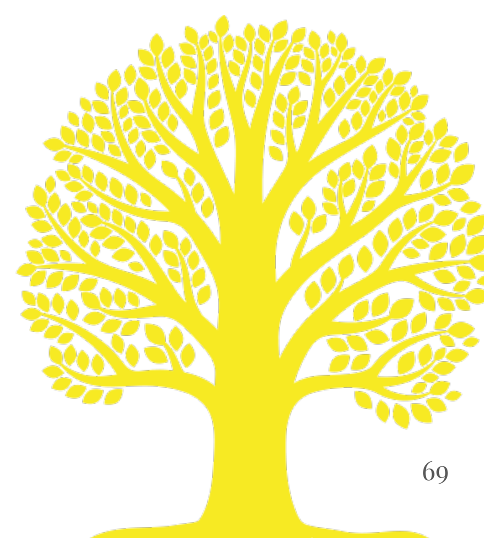
- Work to key dates – have a departmental schedule

- Share key dates with students – ensure they have them in a diary and that key dates are on the school website

- Decide on the order of teaching the units – one teacher or two teachers.

Teacher A	Pure 1 followed by Pure 2
Teacher B	Mechanics 1 or Statistics 1 or Dec Maths 1

Teacher A	Pure 1
Teacher B	Mechanics 1 etc
Teacher A and B	share Pure 2



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Support from Pearson

Scheme of Work – Overview – Translate into weeks at school

- Available for both IAL and International GCSE on the website
- Overall planner with detailed lesson plans
  - Objectives of lessons
  - Teaching points
  - Common misconceptions and errors
  - Opportunities for problem solving

Support overview

Free Support

Getting Started Guide & Scheme of Work	Getting Ready to Teach Events	Subject interpretation of transferable skills
Subject Advisor	**ResultsPlus**	Regional Support Manager

Additional support for selected subjects

Curriculum Matched Publishing	Lesson plans	Exemplar Marked Responses
Topic booklets & Subject guides	Additional SAMs	**examWizard**

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https://www.mathsemporium.com/

Contact your dedicated Subject Advisor

Subject Advisor details

Your subject advisor is **Graham Cumming**
Phone: **+ 44 (0)20 7010 2174**
Twitter: **@EmporiumMaths**
Email: Teachingmaths@pearson.com

Sign up for monthly newsletters from https://qualifications.pearson.com/en/forms/keep-updated-on-pearson-edexcel-qualifications.html to stay on top of qualification updates, training, course materials and industry news and https://qualifications.pearson.com/en/forms/sign-up-international-online-subject-expert-panels.html to see what other teachers are thinking and doing.

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Connecting with other professionals

Pearson International Schools Community

Connect with international teachers around the world

- Connect with other teachers working in international schools and join groups who have shared interests, subjects or location
- Read topical news and articles and share your own
- Advertise jobs at your school or find job opportunities
- Download free resources
- Sign up for events.

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ALWAYS LEARNING